The Irrelevance Of Relevance

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Abstract: The lack of a theory of relevance in the current state of the art of informal logic has often been considered regrettable, a gap that must be filled before the Relevance-Sufficiency-Acceptability model can be considered complete. I wish to challenge this view. A theory of relevance is neither desirable nor possible. Informal logic has got by perfectly well with relevance judgments that are unanalysed and can get by equally well with relevance judgments that are unanalysable. Criticism of theories of relevance, for example in Woods (1992), is deflated.

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1. The function of relevance judgments

We wish to capture the distinction between good arguments and bad arguments. One traditional view has it that this distinction maps on to the valid/invalid distinction in formal logic: good arguments are those that are logically valid and bad arguments are those that are logically invalid. Informal logicians have attacked this as leading to false negatives and false positives. Instead of validity they offer three joint criteria for argument goodness: the premises must be relevant to the conclusion, they must be sufficient for the conclusion, and they must be acceptable. This is the famous RSA model [for a brief recapitulation of
the RSA model see van Eemeren, Grootendorst, and Henkemans et al (2009: 178-79)].

A false negative might be thought to arise when the premises are good reasons to believe the conclusion to be true but do not establish it conclusively. This is a good argument but is deductively invalid; therefore, goodness is not reducible to validity. Or so a familiar line of argument goes. An equally familiar line of counter-argument says that although the premises do not establish the conclusion conclusively, it can be considered as an enthymeme that would establish the conclusion conclusively. One must be careful not to misunderstand what is being claimed in this counter-argument. It is not being claimed that the enthymeme actually does establish the conclusion conclusively—one cannot make certain what was uncertain without the addition of new information—but only that (supposing the premises themselves to be certain) the degree of certainty of the conclusion is equal to that of the logical minimum. For instance,

\[ p \\
q \\
r \]

is invalid but can be made valid by the addition of the unexpressed premise

\[ (p \land q) \supset r. \]

Given that we are certain that \( p \) and that \( q \), our certainty that \( r \) relative to \( p \) and \( q \) is our certainty that \( (p \land q) \supset r \), i.e., our certainty that the antecedent and consequent are true in this particular situation.

The qualification is important. Essentially, what I am arguing is that deductively valid arguments be treated analogously to statistical syllogisms, that is to say, we do not detach the conclusion from the premises; it is only relative to \( p \) and \( q \) that our certainty that \( r = \) our certainty that \( (p \land q) \supset r \). We cannot say in an absolute sense what the probability of the conclusion is. Deductive entailment is a relation between premises and a conclusion and does not allow you to say anything unconditionally about either terms of the relation in isolation of the other, including anything about whether their individual probabilities are above some absolute threshold. Hence, on this conception, the fact that the probability of the conclusion may be below (because it is the numerical product
of) the probabilities of the premises is a red herring. The alternative conception where unconditional statements are made about the premises and the conclusion is not deduction itself but a corollary of deduction that is true under very particular epistemic circumstances. It is not a logical feature or principle that licenses detachment but an epistemic feature, namely that the premises are always better known than the conclusion and hence in the choice a valid argument gives you between accepting a conclusion and rejecting a premise it is always the conclusion that is accepted, rejection of the premises having been ruled out in advance. Were we to be certain that \((p \land q) \supset r\) we would be certain that \(r\) and we could detach \(r\). This, however, is an epistemic feature of a special case and is not implied by the mere fact that the argument is valid or sound; the norms of logic only tell you to maintain consistency, not specifically to accept a conclusion. However, we may be certain of the conditional only because we have strong evidence for the antecedent and for the consequent (or even, perhaps, because we are certain that the antecedent is false or that the consequent is true) and this, clearly, does not necessarily mean that the antecedent is a strong reason for the consequent in general. We may be quite certain that “If the roads are wet then it has been raining” if we have strong evidence that the roads are wet and strong evidence that it has been raining, even though we know that there are exceptions if we generalize this result and even if we do not generally take the fact that the roads are wet as itself a strong reason to believe that it has been raining or even (bizarrely) believe that wet roads are negatively correlated with rainfall. This is what I meant by the qualification in this particular situation.

This being so, there is no false negative, but the danger is that we have purchased the absence of false negatives at the price of there being no negatives at all. The interpretative principle that all arguments should be supposed to be good wherever possible and interpreted accordingly is ‘apocalyptic’ if good means only that it be deductively valid. Any argument at all can quite trivially be made valid by the addition of an unexpressed premise, which would mean that every argument is good, and there are no bad or invalid arguments at all. What we get instead is a fool-proof way of generating false positives. The problem then is to find another criterion by which we may rule out false positives, and the solution might appear to be the relevance condition of the RSA model.

Arguments made valid in this way but whose premises are irrelevant to the conclusion would be called bad. This does not in itself mean that some of those instances that are bad cannot be
ruled out by appeal to logical criteria, e.g., because the premises are logically contradictory or the conclusion logically tautologous or the inference is circular. What we are interested in is those that are not so ruled out, that is to say, those arguments that are bad for no other reason than that a non-truth-functional relation fails to hold. Walton (2008: 140) gives:

Roses are red  
Einstein was a genius  
If roses are red then Einstein was a genius

In claiming that this is not valid Walton gives the “if … then …” a reading that is not a material implication. A relevance judgment must be a judgment about this relation. Here, we judge that the argument is bad because of “a failure of relevance in the conditional conclusion.” I am not so sure about Walton’s way of making his point. I am quite happy to treat the argument as valid and its conclusion as true. Indeed, in this particular situation the conditional is true. Despite this, I would uphold Walton’s verdict that this is a bad argument and that the badness is due to a non-truth-functional relation failing to hold (which does not need to be interpreted as a non-truth-functional conditional failing to be true). The relevance criterion is meant to be an additional condition on argument goodness and not a constraint on validity.

If a theory of relevance is to succeed where validity failed it must satisfy the following trivial-looking constraints:

(AC1:) A theory of relevance should not imply that relevance is (or is approximately) apocalyptic.

In the paper “Apocalyptic Relevance” John Woods defines a theory of relevance as apocalyptic if it has the result that nothing is relevant to anything or that everything is relevant to everything (Woods 1992: 189). His conclusion is that propositional theories of relevance are apocalyptic by focussing on one such theory. Obviously, if he is right, then we seem to be no better off than before. The next section of this paper will discuss whether Woods’s charge is justified.

To this I would add the further constraint:

(AC2:) A theory of relevance should be able to “slot into” the Relevance-Sufficiency-Acceptability model for the goodness of argumentation.
In a good argument the premises are relevant to the conclusion (or equivalently, the antecedent of the logical minimum is relevant to its consequent). Put epistemically, the premises are good reasons to believe the conclusion to be true; they have probative force.

Satisfying AC1 is a minimum requirement for satisfying AC2, since an apocalyptic theory where everything is relevant to everything cannot distinguish good arguments from bad. The relevance/irrelevance distinction must capture or contribute to capturing the good/bad distinction. It is this that motivates the desire for a theory of relevance.

This is not the same as capturing so-called fallacies of relevance, which are simply deductively invalid arguments. Now, an invalid argument can always be made valid by adding as a premise the logical minimum, as already shown. However, there are cases where it is clearly wrong to add the logical minimum, and this can be because we as interpreters can see that the arguer is not committed to the logical minimum or is so committed but for bad reasons.

An example of the former might be an argumentum ad misericordiam. Here, the arguer is not charitably interpreted as being committed to the conditional that she deserves an A in a test because she would be unhappy otherwise; she knows perfectly well that her potential unhappiness is irrelevant to the truth of what mark she deserves, or so one would hope. There is, then, no disagreement on this point between her and her target, and indeed it seems inappropriate to evaluate her argumentation as if there were. Her plea is rather that in these circumstances truth should be subordinated to a different value. She is not arguing at all, therefore it is possibly uncharitable to interpret her as arguing fallaciously, but if she is taken to be providing an argument, deductive validity alone tells you why the argument is bad.

An example of the latter might be an equivocation. What an arguer might take as evidence for a claim when an ambiguous term is interpreted one way might only support the claim when the ambiguous term is interpreted in the other way. Since the arguer herself takes the terms in the same way in both premises she is here committed to the logical minimum, yet an interpreter, seeing that this commitment is based on a bad reason, would not allow the arguer this and would decide that the arguer has committed a fallacy although from the arguer’s own point of view the reasoning is perfectly sound.

In other words, the interpreter decides to interpret the argument without the logical minimum, i.e., in such a way that it is fallacious, rather than interpreting it and then evaluating it.
The processes of interpretation and evaluation are, I believe, dynamically interrelated in this way.

Let us return to Walton’s example. It is not clear that the premises are irrelevant to the conclusion, such as it is. The same seems to apply to an argument like

Roses are red
If roses are red then Einstein was a genius
Einstein was a genius

What does the badness of this argument consist in? Not in invalidity, because it is valid. Not in unsoundness, because it is sound. If it is not that the premises are irrelevant to the conclusion either, then what? It consists, it seems to me, in the badness though not the falsity of the conditional premise; saying that the premise “Roses are red” is irrelevant to the conclusion “Einstein was a genius” is a slightly misleading way of saying that the antecedent of the conditional premise is irrelevant to its consequent; this conditional is not a good warrant for drawing the conclusion that Einstein was a genius even if it so happened that roses are always and everywhere red. The redness of roses is not a good reason to believe that Einstein was a genius, but the wetness of roads is a good reason to believe that it has been raining, and our theory of relevance should be able to explain why.

The diagnosis seems to be this. From $p \land q$ (or even from $\neg p$ alone or from $q$ alone), $p \supset q$ logically follows, yet despite being truth-preserving it does not constitute a good warrant, and this seems to be because it does not generalize (in some sense to be determined). It is generally agreed that commitment to the conditional should not depend on prior commitment to the consequent [i.e., from $q$ alone—the accounts of Hitchcock and Bermejo-Luque discussed in Hitchcock (2011) both emphasize this point]. But maybe this result has been overstated. If we know that in the logical model of the actual world $p$ and $q$ are both true then it seems to me that believing $p$ is a reason for believing $q$ since it is a reason (however weak) for believing we occupy a $p$-and-$q$-world even in the case illustrated above. Also, one cannot rule out in advance a context where $p$ is relevant to $q$. Relevance should turn out to be relative to context. However, I agree that to be a really good warrant more is needed—a conditional must be good (both true and relevant) in at least those possible worlds where the antecedent is satisfied, e.g., where roses are red.
Judging that \( p \) is relevant to \( q \) then roughly amounts to judging that \( q \) is true in more \( p \)-worlds than \( \neg p \)-worlds, or going back to our previous example, if \( p \) and \( q \) are judged to be irrelevant to \( r \) then we have no general reason (though we may have particular reasons) to believe the associated conditional, and because we are not justified in believing this we are not justified in believing the conclusion. When we argue sincerely for a conclusion we are committed to the truth of the associated conditional (which does not necessarily mean that we are certain of it) and to the claim that our belief in the conclusion is based on the reasons referred to in the conditional. Relevance is a condition on inferential justification over and above mere logical deducibility designed to rule out false positives, i.e., those cases where the conditional is not warrant-conferring.

I wish to compare this with the view recently given by David Hitchcock (2011). He also rejects deductive validity as sufficient because it allows ex falso quodlibet and ex quodlibet verum and suggests that this can be remedied if we add as a condition that “there is a generalization of an argument's associated material conditional . . . that is necessarily true, even though it can have an instance with a true antecedent and can have an instance with an untrue consequent” (Hitchcock 2011: 195).

There are several interesting features of this analysis. One is that there is no single generalization. We can generalize on any “content expression”: subject terms, predicate terms, or both. Thus, the premises are relevant to the conclusion if one of these generalizations satisfies the given condition even if other generalizations do not. Another feature is that when the condition is satisfied there must be topical overlap between the premises and conclusion, for were the antecedent true and the conclusion false for the same instance this would be ipso facto a counter-example, which is to say that the generalized conditional is not necessarily true, or to put it another way, the argument is deductively invalid, having true premises and a false conclusion.¹ So-called topical relevance is not itself and

¹ For instance, “Socrates is human, therefore Socrates is mortal” exemplifies relevance because we can generalize on the predicate term to give \( F(Socrates) \supset G(Socrates) \) and there is some \( F \) for which \( F(Socrates) \) is true, some \( G \) for which \( G(Socrates) \) is false, and some \( F \) and \( G \) (namely “human” and “mortal” respectively) for which it is impossible for \( F(Socrates) \) to be true and \( G(Socrates) \) to be false. “Socrates” is here the topical overlap. In contrast, “Socrates is human, therefore Plato is mortal” does not exemplify relevance because the fact that \( F(Socrates) \) is true and \( G(Plato) \) is false for some \( F \) and \( G \) means that there is no \( F \) and \( G \) that makes \( F(Socrates) \supset G(Plato) \) impossible; there are no logical dependencies between Socrates’ F-
nor does it guarantee the kind of relevance required for a good argument. In fact, the following bad argument given by Hitchcock has topical overlap and satisfies the condition above:

\[
\begin{align*}
&\text{Napoleon ruled France} \\
&\text{Napoleon was born in Corsica} \\
&\text{Napoleon was imprisoned on Elba}
\end{align*}
\]

Here “Napoleon” is the topical overlap and there is no substitution where the premises are true and the conclusion false. To cope with such cases of irrelevance Hitchcock (2009: 17-19) adds that the generalization must support counterfactuals. If someone other than Napoleon had ruled France and been born in Corsica, it would not follow that they were exiled to Elba. For the conditional to be necessary on this revised conception means that it does not have any actual or counterfactual instance with a true antecedent and a false consequent (Hitchcock 2011: 202). Whether a conditional is necessary on either conception seems to be only an intuitive judgment, however (Hitchcock 2009: 9).

Now, I agree that Hitchcock succeeds in ruling out \textit{ex falso quodlibet} and \textit{ex quodlibet verum} but I would deny that the condition that does so is non-logical despite the fact that Hitchcock takes his definiendum to be \textit{non-logical consequence}. Logical criteria can rule out these cases (especially if premises are actually contradictory or the conclusion tautologous) even if validity does not, and Hitchcock’s condition is a plausible candidate for such a criterion. These are not the most interesting cases (cases where it is a non-truth functional relation that fails to hold), as already said. Also, it seems that we do not always want to rule them out, as Woods will be shown to argue later and Hitchcock (2009: 13) himself points out. Hitchcock’s way around this problem is to introduce content expressions with its resulting topical overlap. However, I aim to show later in the paper that topical overlap is as slippery a notion and equally as reliant on intuition as judgments of necessity, for of any two propositions it is logically necessary that there be a proposition that topically overlaps and is relevant to each of them, so if relevance is transitive (which may of course be denied, but I will

\[\text{ness and Plato’s G-ness such that the former can rule out the possibility of the latter being false. Similarly, if we generalize instead on the subject term we get Human}(x) \supset \text{Mortal}(y),\text{ and provided that } x \neq y \text{’s being human is always logically compatible with } y \text{’s not being mortal.}\]

2 Relevance logic is an attempt to design a logic that rules out paradoxes of material implication such as \textit{ex falso quodlibet} and \textit{ex quodlibet verum} and does not provide a theory of relevance in the sense this paper discusses, viz., an analysis of the concept of relevance (Woods 1992).
argue that it is) we get the by now familiar apocalyptic result that everything is topically relevant to everything.

Nevertheless, topical relevance is the key, since any comparison against counterfactual scenarios or possible worlds must take something as a fixed point from which the closeness of such worlds is to be judged. I do not see why it has to be a generalization of the associated conditional and not the associated conditional itself that is to be judged. Although I also say that the conditional must “generalize” it is possible worlds and not content expressions that I mean to quantify over. Thus, Hitchcock’s appeal to counterfactuals does not work out the same as my appeal to possible worlds, nor does it give the same verdict in the Napoleon example. Since there is topical relevance the premises are relevant to the conclusion because if the premises are true it proves at least that Napoleon exists, but it is only weak on the grounds that there are nearby possible worlds in which Napoleon was not exiled to Elba. However weak, topical relevance is probative relevance.

The “apocalypticity” of relevance (whether probative or topical) has the apocalypticity of justification as a consequence; any invalid argument can be trivially made valid by the addition of the logical minimum, and if also everything is relevant to everything then no argument at all is bad. The criteria of validity and relevance do not rule out anything at all, either individually or jointly.

2. Theories of relevance

Most theories of relevance are propositional, that is to say, they construe relevance as a relation between propositions. Many theories make it a triadic relation where the third term is something like the context, common knowledge, or the beliefs (or expected beliefs) of an audience. Each submits to the same formal treatment, so these variants need not detain us, but is should be noted in passing that the last especially introduces a rhetorical dimension to the notion of relevance; if in my argumentation I support my thesis with propositions that I do not suppose my audience to share or try to motivate them by appealing to values that they do not share then it seems valid to criticize my argumentation for being irrelevant while for another audience, my argumentation would be completely relevant. The function that the reason that I give is supposed to perform is that, together with other information I expect the listener or audience to provide for themselves, it promotes adherence to the thesis. I agree that this is a prototypical case of relevance, but I
have my doubts whether the portion italicized is actually a necessary condition, as will become clearer; deviation from the prototypical case is possible and yet may still be a case of relevance.

The question we must now address, through Woods’s critique of a theory of relevance advanced by Sperber and Wilson, is whether propositional theories are apocalyptic. If a theory satisfies (AC1) there is at least some reason to suppose it satisfies (AC2). To put Sperber and Wilson’s theory into context it should be noted that they are concerned with the goodness of linguistic interpretation rather than the goodness of arguments and when they speak of context they do not have in mind the unexpressed premises of an audience to be persuaded but of resolving semantic ambiguity through appeal to the circumstances in which something is spoken; yet, since interpretation is the product of a (non-deductive, in their case) inference it is analogous to the cases that concern us. In particular, Sperber and Wilson express the common intuition that the thesis should not be inferable from the context alone but only together with the uttered proposition in order for that proposition to be relevant or, in their words, to have a contextual effect where such an effect occurs if it “strengthens or reinforces a belief contained in that context, when it contradicts a belief contained in that context . . . or when it licenses implications” (Woods 1992: 190).

It is with this “licenses implications” that Woods is mainly concerned. It boils down to the condition that P is relevant to Q in context C if P and C together non-trivially imply Q but neither P nor C on their own non-trivially implies Q. Woods argues that this has apocalyptic consequences and strongly implies that this result can be generalized to all propositional theories. Suppose, Woods says, that C = {Q} and that P and Q are logically independent. This satisfies the given condition to give a false positive result, for

1. \( P \land C \) non-trivially implies Q (by the specification of C and \( \land \)-elim),
2. P does not non-trivially imply Q (by the independence condition),
3. C does not non-trivially imply Q (for Q does not non-trivially imply itself because \( Q \vdash Q \) is not an elimination rule but a redundancy rule).

Non-trivial implication is defined by Sperber and Wilson (Woods 1992: 190) as:
Non-trivial implication. P logically and non-trivially implies Q iff when P is the set of initial theses in a derivation involving only elimination rules, Q belongs to the set of final theses.

The result is that every P logically independent of Q is relevant to Q in C, which is close enough to apocalypse as to fail to satisfy AC1. (Woods 1992: 191)

It is not entirely clear why P∧C non-trivially implies Q as it says in step 1. Whatever follows from the application of an elimination rule follows trivially, according to Sperber and Wilson. So, applying ∧-elim to P∧C shows P and C to be trivially implied. Even though C = {Q}, strangely Q is non-trivially implied while C is trivially implied. This already seems to have the implication, later objected to by Woods, that both trivial and non-trivial implication are not closed under logical equivalence, for surely C is logically equivalent to Q if C = {Q}. Let us accept this. The point then seems to be that neither P nor C on their own non-trivially implies Q, as 2 and 3 say. Since C always implies Q by itself but non-trivially, it follows that the conjunction of C with anything that does not itself imply Q also non-trivially implies Q. So, as Woods argues, any such proposition is relevant to Q in C. I will call this argument FP to indicate that the result is a false positive.

Woods brings forwards two more objections. These are false negatives.

The first is that we would normally say that P is relevant to Q if P entails Q, but this does not follow on Sperber and Wilson’s theory. This contradicts the intuition expressed above, P being relevant to Q in context C even if P entails Q on its own and C has no role to play.

Can we simply get rid of the condition that P is not relevant to Q if Q can be inferred from P alone? Hitchcock (1992: 260) defines premise relevance as: “A premise is irrelevant if it cannot ineliminably be put together with other at least potentially accurate information to produce a set of premises which is sufficient to justify the conclusion. Otherwise it is relevant.” He goes on to claim that a premise is not relevant to itself because it cannot be used to establish itself but what he does not say is that a premise is not relevant to a conclusion that it establishes on its own; it is consistent with Hitchcock’s definition that P is relevant to a Q that it entails.

The second objection is that if P explains Q then it follows on Sperber and Wilson’s theory that P is not relevant to Q. Woods’s (1992: 197) point here seems to be that the context here contains the explanandum Q and wherever Q does appear it...
trivially implies itself. This seems to contradict what Woods says in step 3 of FP where \(\{Q\} \vdash Q\) is said to be a non-trivial implication. Woods would perhaps still be consistent by claiming that although \(\{Q\} \vdash Q\) is non-trivial \(\mathbf{C} \wedge Q \vdash Q\) is trivial (i.e., \(Q\) is only one member among others of the context \(\mathbf{C}\)), and perhaps this can be argued on the grounds that \(Q\) follows from \(\mathbf{C} \wedge Q\) but not from \(\{Q\}\) by an elimination rule. Is, then, \(\mathbf{P} \wedge \mathbf{C} \vdash \mathbf{Q}\) also a trivial implication when \(\mathbf{C} = \{Q\}\)? If so, how is this not inconsistent with step 1 of FP? I am confused by Woods’ argument here but accept the moral.

Consider also this similar case (not given by Woods). Suppose that there are two members of \(\mathbf{C}\) such that \(\mathbf{C}_1 \wedge \mathbf{C}_2\) non-trivially implies \(Q\) and \(\mathbf{C}_1 \wedge \mathbf{P}\) non-trivially implies \(Q\). Now \(\mathbf{C}\) implies \(Q\) on its own so \(\mathbf{P}\) is not relevant to \(Q\) in \(\mathbf{C}\). Should it be? Consider also that \(\mathbf{P}\) is certainly relevant to \(Q\) in \(\{\mathbf{C}_1\}\). Why should adding \(\mathbf{C}_2\) to the context destroy the relevance of \(\mathbf{P}\)?

These are cases—contrary to the intuition expressed above—where \(\mathbf{P}\) is relevant to \(Q\) in context \(\mathbf{C}\) even if \(\mathbf{C}\) implies \(Q\) on its own. Here it is \(\mathbf{C}\) and not \(\mathbf{P}\) that is ineliminable yet we would still say that \(\mathbf{P}\) is relevant to \(Q\) in \(\mathbf{C}\). Thus it is questionable whether it is necessary for \(\mathbf{P}\) to be ineliminable in order to be relevant. Note though that Hitchcock refers to ineliminability together with some other information, not necessarily all the information that is contained in the context. Thus Hitchcock’s definition does seem to say, correctly in my view, that \(\mathbf{P}\) is relevant in this situation. It also seems to say that \(\mathbf{P}\) is relevant to \(Q\) when \(\mathbf{P}\) explains \(Q\) (i.e., \(Q\) is in \(\mathbf{C}\)) as long as the other elements of the non-trivial implicans of \(Q\) are in \(\mathbf{C}\).

However, this also seems to have a false positive. Suppose that in the last example \(Q\) is \(\mathbf{P}\) or something trivially implied by \(\mathbf{P}\). Because \(\mathbf{P}\) can ineliminably be put together with some other information as to imply everything \(\mathbf{P}\) trivially implies, \(\mathbf{P}\) is now relevant to itself and everything it trivially implies. The distinction between trivial and non-trivial implication breaks down. Nor do we need to suppose that \(\mathbf{C}\) implies \(Q\) on its own. Suppose \(\mathbf{P}\) is \(p\) and \(\mathbf{C}\) is \(q\). Now we can imply \(p\) trivially from \(\mathbf{P}\) alone by redundancy, or from \(\mathbf{P}\) and \(\mathbf{C}\) non-trivially by \(\wedge\)-Intro followed by \(\wedge\)-Elim. \(\mathbf{P}\) is ineliminable from this derivation. This result does not seem to follow for Sperber and Wilson since although \(\wedge\)-Intro is not an elimination rule and hence \(p \supset p\)

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\(^3\) Here there are two paths to the derivation of \(Q\), in only one of which is \(\mathbf{P}\) ineliminable. Nevertheless, this is all that is needed for Hitchcock’s definition to have the correct result that \(\mathbf{P}\) is relevant to \(Q\). One can think of this as having convergent arguments for \(Q\), or as \(\mathbf{P}\) having the contextual effect in \(\mathbf{C}\) of strengthening or reinforcing a belief (namely, \(Q\)) contained in that context.
would be a non-trivial implication, because of the trivial implication of \( P \) by \( C \) alone we do not have to say that \( P \) is relevant to itself. Even so, it would be preferable if there were a way to define non-trivial implication that did not rely on this additional condition on relevance. I will consider another way discussed by Woods and then suggest a way of my own.

This leads us back to Woods’s first objection that I called FP. It seems to me that the moral is how counter-intuitive the non-triviality of \( \{Q\} \vdash Q \) is. Woods considers a different way of defining non-trivial implication and shows that it also leads to apocalypse despite having the result that \( \{Q\} \vdash Q \) is trivial. Originally the implication is construed as trivial if it follows by the application of only elimination rules, but Woods considers instead whether we should not say that it is trivial if it follows by what Sperber and Wilson term an analytic rule rather than by what they term a synthetic rule, where a synthetic rule requires two inputs. This blocks step 1 because \( \wedge\text{-elim} \), says Woods, an analytic rule since \( P \wedge C \) is a single input. Incidentally, \( \{Q\} \vdash Q \) seems likewise to be an analytic rule and step 3 also seems blocked, although Woods does not remark on this.

Woods rejects this move because the same reasoning that makes \( \wedge\text{-elim} \) analytic makes the licensed implication similarly analytic. If \( P \wedge C \) is a single input and \( Q \) has to follow from \( P \wedge C \) by a synthetic rule in order for the premise to be relevant to the conclusion in that context, then the consequence is the conversely apocalyptic one that nothing is relevant to anything (Woods 1992: 192).

Is the distinction between a synthetic rule and an analytic rule only a notational distinction? Is \( \{P, C\} \vdash Q \) always and everywhere the same as \( P \wedge C \vdash Q \)? By stipulating that the premise and the context are to count as separate inputs we could make the licensed implication synthetic, and conversely any synthetic rule has a logically equivalent analytic rule formed by the simple expedient of conjoining the premises into a single logical expression. We could, Woods speculates, make conjunction an optional rule—that is to say, from \( \{P, C\} \) inferring \( P \wedge C \) is permitted but not obligatory—and by distinguishing in this way the inferential relation \( \{P, C\} \vdash Q \) from \( P \wedge C \vdash Q \) argue that \( P \) implies \( Q \) is licensed in the first but not the second. According to Woods (1992: 193-95), this has the undesirable consequence that licensed implication and hence relevance are not closed under logical equivalence (but recall that we seemed to have this result anyway in step 1 of FP). Even if we somehow manage to avoid or to accept this undesirable consequence, we would not avoid apocalypse because \( \wedge\text{-elim} \) can also be con-
structured as a synthetic rule, viz. \{P, C\} \vdash C. The moral is: whatever makes \&-elim analytic makes licensed implication analytic, and whatever makes licensed implication synthetic makes \&-elim synthetic. Either everything is relevant to everything, or nothing is relevant to anything.

Triviality has been construed so far with respect to elimination rules and synthetic rules, and both were found to have apocalyptic results. What Sperber and Wilson are, I think, trying to express in their notion of triviality is this. In some valid arguments it is only in a very trivial sense that the conclusion can be said to follow from the premises, the most trivial being \(p \vdash p\). I suggest that \(p \vdash \neg \neg p\) and \(\{p, q\} \vdash p \& q\) are only superficially less trivial and for the same reason as \(p \vdash p\): the conclusion only asserts again what was asserted in the premises. However, \(\{p, p \supset q\} \vdash q\) and \(\{p \lor r, [(p \supset q) \& (r \supset q)]\} \vdash q\) are not trivial, and for the converse reason: the conclusion asserts something not asserted in the premises. Clearly the triviality is not a matter of elimination rules, since trivial cases used introduction rules and both non-trivial cases used elimination rules. Rather, triviality follows when the rules used preserve unassertedness and non-triviality follows when they do not preserve unassertedness. Rules that do not preserve unassertedness are \supset\-elim and \lor\-elim, because when these are applied some expression that was unasserted becomes asserted, e.g., in \(p \supset q\) both \(p\) and \(q\) are unasserted but when \(p\) is asserted as a premise and \supset\-elim is applied then \(q\) also becomes asserted. It is in this sense that such rules require two inputs—it requires two assertions.\(^4\) In this sense the rules are synthetic (but so is \&-Intro).

Does this avoid apocalyptic results? On the Sperber and Wilson definition of relevance, now that \{Q\} trivially implies \(Q\)—blocking step 3 in FP—\(P\) is not relevant to \(Q\) in \(C\) when \(C = \{Q\}\). We no longer have this false positive, but we still have the false negatives, so although AC1 is satisfied, AC2 is not. The reason, to re-iterate, is that there is a clash of intuitions; our original intuition that \(P\) is not relevant to \(Q\) in \(C\) if \(P\) or \(C\) non-trivially implied \(Q\) on their own seems, on reflection, to have exceptions, and there is no set of formal conditions that those exceptions uniquely satisfy. We are not forced to say that \(P\) non-trivially implies itself in any \(C\), \&-Intro and \&-Elim both preserving unassertedness and thereby making the implication trivial. But if \(C\) is such that it implies \(P\) non-trivially we still seem

\(^4\) The conditional itself is asserted even though the propositions embedded in it are not. See Botting (2011a) for a further explanation of unassertedness.

forced to say that P is relevant to itself, and this applies to any such P. The result is that everything non-trivially implied by a context is relevant in that context to itself and to everything it itself implies.

Propositional theories all seem to have unwanted consequences. What moral should we draw from this? You might continue the search, but these consequences seem quite general. You might want to reject AC1 or AC2. Or one might decide to keep AC1 and AC2 and argue that the concept of relevance is incoherent because these constraints cannot be jointly satisfied. I want to draw a slightly different conclusion.

3. The irrelevance of relevance

First, I want to raise a different objection to theories of relevance. I want to suppose ex hypothesi that there is a set of formal conditions that are individually necessary and jointly sufficient to establish that a premise is relevant to the conclusion. In other words, I want to suppose that a theory satisfying the constraints is out there awaiting discovery. Making a relevance judgment consists, then, of going through these conditions and judging whether they are satisfied. Do we then have a complete and formal procedure for evaluating the argument, as informal logicians desire? I think not, and this is because any formal analysis will make further claims about the components of the relation which may themselves be questioned on grounds of (a) whether we are justified in believing them, and (b) whether they are relevant to the conclusion that P is relevant to Q in C, for concluding this because it satisfies sufficient conditions for relevance is itself an instance of *modus ponens* and being an instance of *modus ponens* is not in itself sufficient to show the argument to be good—one must make a relevance judgment in order to draw a conclusion about relevance.

(a) is a general problem for any judgment that is meant to have a justificational function: one tries to justify some proposition by making judgments about the truth of the premises and the validity of the inference, for example, but if you then provide a theory of truth or of inference then the result will be more claims that will themselves require judgment and about which one may have doubts—there will be an epistemic regress. Even if these claims are knowable *a priori* this does not rule out the possibility of errors in performance and this will introduce doubt, since it would be strange to say that you know something to be true while believing that you may have made a mistake in
its derivation (witness Descartes’ rejection of mathematical truths as something that is indubitably known).

In a recent paper (Botting 2011b) I made this claim with respect to modus ponens itself and supposed that the person using it justified his use of it not, e.g., in terms of its self-evidence or by logical axioms, but on the *a posteriori* ground that he had been taught it by a teacher who, unknown to him, was unreliable. I then asked whether the reasoner was justified in believing in his conclusion. The answer was no: the reasoner was lucky that the unreliable teacher was right with respect to modus ponens being a truth-preserving rule of inference. My conclusion was that there must be another kind of justification and that this involved understanding the attribute involved in the antecedent of the conditional and, taking the subject of the antecedent as the *de re* intentional object, ascribing this attribute to that object. Even if the reasoner was unjustified in his application of modus ponens, provided he possessed this *de re* attitude, that is to say, provided he understood and was successful in ascribing the attribute of redness to roses, this justified the conclusion and he genuinely knew, e.g., that roses are coloured.

Without a *de re* belief the reasoner is in a position analogous to that of the occupant of the Chinese Room who by manipulating the symbols he is given following rules he is given is able to make statements in Chinese that, conceivably, he knows to be true. But his true belief is not about whatever the Chinese statement is about, because he does not understand Chinese or what the content of the statement is, but about a particular sequence of symbols. All of his beliefs are *de dicto*. Applying rules of inference is analogous, and having conclusive proof of a conclusion from known premises does not make the content of the conclusion a case of knowledge. For example, applying modus ponens to “Roses are red; if roses are red then roses are coloured” to conclude “Roses are coloured” does not amount to knowledge even if using this rule is justified for the reasoner unless redness is understood (which involves its being a colour and is not a matter of being able to give a set of conditions) and ascribed *de re* to roses. To know something, then, is not to assent to a proposition but must involve original intentionality, and it is indifferent to the possibility of errors in performance or the unjustified use of rules. But if we were ask what redness means and only accept as an answer a theoretical definition of redness in the form of a set of sufficient conditions for redness, i.e., as would be given by a theory of the semantic representation involved, then this would be counter-productive, since as soon as you gave one it would not be able to play the
justificational role required because of the implicit regress, and instead of a *de re* belief about roses one has *de dicto* beliefs about some conditions or rules.

I suggested that there was no philosophical theory of meaning or mental representation, that none was even possible (Botting 2011b), but perhaps what I should have said is more modest: it is fruitless to search for or to have a theory of meaning because, supposing one to be possible, it would still not do what you want it to do.

This is basically my position also with regard to relevance. When I speak of the irrelevance of relevance I do not wish to be taken as implying that it is not necessary to make relevance judgments; what is irrelevant is a theory or formal analysis of relevance. The idea that such judgments need a theory behind them to make them respectable is, I believe, entirely false. On the contrary, judgments intended to have a justifying function cannot be backed by such a theory without initiating a regress.

Instead of a theory of relevance, what one might attempt to provide is a phenomenological account of relevance judgments, just as instead of a theory of meaning one might provide an account of how it feels to take a class of linguistic or perhaps mental objects to be equivalent in meaning. To the question “What does redness mean?” one can answer only “The same as that” or less strictly “Something like that.” In this way the class of relevance relations is a generalization of the stricter class of meaning-equivalents. This is a purely descriptive account. Slotted into the RSA model it tells us what we are doing when we evaluate an argument as good or bad, but relies on intuition as the guide of whether the argument actually is good or bad. But recall that Hitchcock’s account also relied on intuitive judgments of necessity, and so also, it seems, does Blair’s, who says that “I doubt that such relevance can be analysed—shown to be derived from or reducible to other concepts ... relevance can be explicated—that is, described in ways which enrich our understanding of it” (Blair 1992: 204). He goes on:

Explicating premissary relevance then becomes a matter of explicating the idea of a premise's lending support to a conclusion. What's involved in this idea is a kind of gestalt—a premise set or pattern in relation to a conclusion *simply is perceived as supporting it*. We can ... express our sense of that gestalt by formulating in propositional form what can be termed the warrant for the inference from the premises to the conclusion. This inference warrant makes explicit, or is the ground of, our
belief that our premises are relevant, by making explicit how we take them to link up with the conclusion. (Blair 1992: 216, my italics)

Like Hitchcock’s, Blair’s explication involves the reasoner being committed to an inference-warrant (Blair 1992: 207-208). I would say that a reasoner is committed to the truth of the associated conditional. These differ in that whereas the associated conditional is formed by taking the logical minimum, the inference-warrant is conceived as being both weaker—not claiming that the conclusion follows deductively but instead having a liberal helping of qualifiers—and also as having more content. But this is a result of the mistaken belief that deductive arguments cannot handle premises with lesser degrees of certainty or inferences whose strength is less than conclusive.

A phenomenological account of relevance judgments cannot be essayed here and must be reserved for another paper. What I would say is that an account that is purely cognitive will not do the job—lexical items must be desired to remain meaning-invariant through time, and that brings in the complicated issue of what consciousness of time consists. I would argue that it is a structural feature of desire that it is projected into the future, and it is from this that we get our temporal concepts of succession and simultaneity, and it is from these that we get our concepts of synonymy and substitutability. Synonymous and substitutable lexical items are characterized by the desire that they be used simultaneously when they cannot be so in fact since all symbolic processing is linear and successive.

What I have suggested so far is that relevance judgments are judgments about the generalizability of the associated conditional. This is because of subject-matter overlap, for instance, the premise “The Queen lives in England” is relevant in a minimal sense to “The capital of England is London” since its truth establishes that there is such a place as England. This is genuinely probative relevance, though of course weak since one can easily conceive of a world where the Queen lives in England and the capital of England is not London. This can be construed as making two de re attributions to England as intentional object. Obviously having London as its capital and being the abode of the Queen are not related as being red and being coloured are related. However, one can imagine being in an epistemic context where the referring terms are intersubstitutable despite not being synonymous, such that in thinking of “where the Queen lives” one ipso facto thinks of “the country that has London as its capital.” Then on learning that it is true that the Queen lives in England one can conclude that England is the
country with London as its capital. This substitution may not be truth-preserving—the Queen may have recently deceased—but this is not the point. The phenomenology of inference (and I include relevance judgments as a kind of inference) is, in my view, the phenomenology of substitution.

4. Conclusion

In this paper I considered ways to try to capture premise-conclusion irrelevance in a case like

- All roses are red
- If all roses are red then Einstein is a genius
- Einstein is a genius

in a formal analysis. Probably the most completely worked out account is in Sperber and Wilson’s “Relevance.” Although in my alternative account of non-trivial implication I found a way to stop the threatened apocalypse I agreed with Woods that sometimes the non-trivial implication of a conclusion by a premise on its own seemed intuitively to be a case where the premise is relevant to the conclusion. Even more damagingly, sometimes the non-trivial implication of a conclusion by a context on its own seemed intuitively to be a case where the premise is relevant to the conclusion. Yet any loosening of this condition on relevance seemed to have apocalyptic consequences.

My conclusion was that a formal analysis of relevance was probably impossible and I considered then how best to proceed. My strategy for dealing with the apocalypse is deflationary: even if a formal analysis is possible, there are reasons why it is neither necessary nor desirable. This was due to the fact that the relevance judgment was required to play a role in justification that no analysis of relevance could play because such an analysis would lead to epistemic regress. I showed then that relevance judgments were on a par with rules of inference in this respect and I outlined a way in which I proposed to terminate the regress involving the latter with a single de re attitude, at least for a certain restricted set of conditionals like “If roses are red then roses are coloured,” this attitude being the de re belief that roses are red, being coloured being implicated in the property of being red and never explicitly mentioned in the “premises.” The belief that roses are red just is the belief that roses are coloured. In such an attitude the meanings of the terms are unanalysable individually but are obviously related to each
other as to guarantee the truth of the conditional. In the case of relevance judgments we needed two *de re* attitudes and there was no guarantee that the one followed from the other. The relevance judgment then amounted to a judgment that in this particular situation a substitution of one descriptive phrase for another preserved truth, or putting it another way that the sentential functions “___ is P” and “___ is Q” are materially equivalent. The relevance judgment itself, then, is *de dicto*. A general account of inference has not been provided but would be based, I suggested, on a phenomenology of substitution of which the kind of semantic relations embodied in the restricted set of conditionals are ideal, context-insensitive cases.

Relevance judgments are not apocalyptic; it seems quite possible to rule out cases like that above on the grounds of premise-conclusion irrelevancy, where this seems to boil down to topical irrelevancy, i.e., a lack of shared subject-matter. However, although possible it is not inevitable, because just as there is always some premise that can be added to make an argument valid, also it is always possible to come up with some proposition that shares subject-matter with the premise and the conclusion; for any true \( q, p \supset q \) trivially follows, which is to say that believing \( p \) can be a reason for believing \( q \). It may not be what we would normally call a good reason, and we may sometimes back up that statement by saying that \( p \) is irrelevant to \( q \) and back that up further by saying \( p \) does not (if this is the case, but there will almost inevitably be some cases where it is not) share any subject-matter with \( q \), yet there is always some context in which by transitive closure of relevance \( p \) is relevant to \( q \) and, taking a rhetorical point of view, always some potential audience bizarre enough to base their belief that Einstein is a genius on roses being red. From this rhetorical point of view, then, apocalypse *is the correct result*, or perhaps we should say more cautiously that the only reason that everything is not relevant to everything is the empirical fact that reasoners and audiences do not make such relevance judgments, at least not with regard to every such \( q \). *It is only a contingent empirical fact that relevance judgments are not apocalyptic*. Although initially this may be shocking, this seems to me correct and not, as an objector might argue, a good reason to say that relevance is not a transitive relation.

The moral is this. Do not search for a theory of relevance. Study relevance judgments if you like, but ultimately this study is probably better conducted on an empirical basis than on a philosophical basis. But most of all: learn to love and to live with the apocalypse!
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