Assessing the Development of Mathematical Concepts in Preschool Children: Checklists for Teachers

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Abstract

This article focuses on three fundamental mathematical concepts for young children and provides checklists as a means of tracing the development of the three concepts: matching and one-on-one correspondence, sets and classification and order and seriation. The checklists provide a developmental sequence in which these three concepts are likely to emerge that can be used to guide curriculum. As a tool for assessment, the checklists can be used to chart and monitor student performance. By studying young children's sequential development as reflected in these checklists, teachers can plan developmentally appropriate experiences for them. Thus, decision making with regard to necessary resources, classroom setup and questions to be asked for the development of each child can be facilitated.

Introduction

Teachers are continually looking for ways to guide students' understanding of mathematical concepts. Documentation of children's work helps teachers plan developmentally appropriate, and more importantly, individually appropriate activities that help raise children's levels of understanding. Accurate documentation and assessment facilitate decision making regarding setting up the classroom, questions to ask and resources to provide, all of which are critical for effective teaching and learning (Helm, Benkee & Steinheimer, 1997). Contemporary research has demonstrated that young children's mathematical abilities remain largely untapped and that they receive a limited introduction to mathematics before they enter elementary school (Balfanz, 1999). Although it has been suggested that preschool programs needed to expand and deepen the

conceptual knowledge that young children have already developed by 3 years of age (Payne, 1990), in 2000 the National Council of Teachers of Mathematics (NCTM) for the first time emphasized that all preschool children need opportunities to explore their world and experience mathematics. The NCTM included the pre-kindergarten years while defining its vision for mathematics education in schools.

Despite this statement there has been no agreed-on mathematical curriculum for preschool. This makes it difficult for teachers to determine which concepts are appropriate for young children. The identification of mathematical concepts that are developmentally appropriate and which lay the foundation for further mathematical understanding is thus considered an important but missing element at the preschool level. This paper focuses on three fundamental mathematical concepts that form the building blocks of mathematical and scientific knowledge. These concepts are matching and one-on-one correspondence, sets and classification and order and seriation. The development of these concepts allows children to organize and categorize information so that when they start formal schooling they begin to apply fundamental concepts while exploring abstract ideas in both mathematics and science (Charlesworth and Lind, 1999).

The constructivist paradigm based on Piaget's (1952) theory of cognitive development has long provided the theoretical framework for educational practice in which children acquire concepts through active involvement with the environment and construct their own knowledge as they explore their surroundings. Applying this theory to mathematics has led to the use of manipulative materials that enable young children to count, engage in active learning, and develop concepts (Kaplan, Yamamoto, & Ginsberg, 1989). Mathematics learning in preschools should be thought-provoking, include opportunities for active learning, and be rich in mathematical language (NCTM, 2000).

Choosing appropriate tools that can be used for effective teaching and assessment are concerns for early childhood educators, especially at the preschool level. Appropriate assessment implies the use of multiple sources to gather evidence about children's mathematical understanding. It is also critical that the evidence be collected in a systematic manner to facilitate the alignment of curriculum, instruction and assessment (Copley, 2000). This paper provides three checklists as tools for teachers to use in their assessment of the development of fundamental mathematical concepts in preschool children.

Checklists: Tools for Authentic Assessment

A checklist is a list of sequential behaviors and/or skills arranged in a system of categories (Wortham, 2001). Checklists can originate from instructional objectives or developmental sequences. The items can record specific skills or different levels of performance. This makes them particularly effective for documenting mathematical skills and knowledge (McAfee and Leong, 1997). Checklists have also been described as a form of curriculum outline that preschool teachers can use to plan instruction, develop learning center activities and evaluate children's progress (Wortham, 2001).

Once set up, checklists can be modified to fit specific classroom or individual needs. Checklists allow teachers to vary materials based on individual needs. By studying the sequential development as reflected in a checklist, the teacher can provide materials and activities suited to specific developmental levels. Documenting evidence through the use of checklists at different times provides a progress record as well.

Fundamental Mathematical Concepts

Young children's understanding of math and science grows from the development of the same basic concepts during early childhood (Charlesworth and Lind, 1999). The following mathematical concepts are considered central to early development of mathematical and scientific thinking:

- matching and one-on-one correspondence
- sets and classification
- order and seriation

Fundamental Concept # 1: Matching and One-on-One Correspondence

Matching is one of the earliest mathematical concepts to develop and forms the foundation for the development of logical thinking. Prerequisite for matching is the understanding of "the same". Research suggests that 2-4 year old children develop this understanding through the relationships of "more-less-the same" (Gellman & Gallistel, 1978; Brush, 1972). Matching is prerequisite for conservation. By age 4 most children are able to match objects. Matching is the basis for the concept of one-to-one correspondence.

One-on-one-correspondence is a fundamental component of the concept of number. It is preliminary to counting and basic to the understanding of equivalence and the concept of conservation of number. Once children have the basic understanding of one-to-one-correspondence they can apply the concept to high level activities involving equivalence and more or less (Charlesworth & Lind, 1999).

The concepts of Matching (Belong together) and one-on-one Correspondence begins with the ability to match items that are not alike, followed by matching similar items. While matching different and similar items, children begin by matching even sets of objects. A higher level of understanding of the concept of one-on-one correspondence is demonstrated when the child is able to match uneven sets of materials. To assess the child's understanding of matching uneven sets the teacher can ask, "can you find a fork for each spoon?" The children's ability to use words such as "not enough" "too many", etc. while matching items that are alike and different would be an indicator of the highest level of their understanding of the concepts of matching and one-on-one correspondence. Given below is the checklist based on the developmental sequence of the concepts of Matching and One-on-one Correspondence (Table 1).

Table 1 Checklist for Preschool Pre-mathematical Concepts: Matching and One-on-One Correspondence**Name of the Child** _____

Concepts/Stages of Development	Sept-Oct.	Dec.- Jan.	April-May
Matching related items that are not alike			
1. Matches different but related items that are not alike			
2. Matches even sets – with 5 or fewer items			
3. Matches uneven sets – with 5 or more items			
4. Uses appropriate vocabulary while matching sets (e.g., too many, not enough)			
5. Matching similar items			
6. Matches 2 similar items			
7. Matches even sets – with 5 or fewer items			
8. Matches uneven sets – with 5 or more items			
9. Uses appropriate vocabulary while matching sets that are alike (e.g., too many, not enough)			

	KEY TO CHECKLISTS
✓	Demonstrates knowledge of the concept
0	Demonstrates partial knowledge of the concept
X	Does not demonstrate any kind of knowledge of the concept

This checklist can be used for teachers' observations in various contexts. In play, for example, a teacher can ask a child to find one plate for each animal at the table, or ask if there would be enough clean spoons for each teddy bear. By providing familiar, everyday objects (both in even and odd quantities), such as cups and saucers, spoons and forks, shovels and pails or sets of plastic animals, the teacher can identify a child's ability to match items that are alike or not alike. If the child's use of materials does not show a clear pattern, the teacher can ask specific questions as, "Can you find two things that go together?"

Good teachers are careful to use language related to the concept of matching and one-to-one correspondence at every opportunity. For example, in the housekeeping area, or the art area they might ask if everyone has a brush to paint or scissors for a project. Or introducing a matching game at clean-up time, can easily convert it into "math time." By asking the children to place one object in a

container or on a shelf, the children are matching object to object, object to picture, and picture to picture. Since one-to-one correspondence is the understanding that one group has the same number of things as another, the child can see that relationship in sets of materials, for example, each child has a chair, each foot has a shoe, each hand has a mitten etc.

The use of children's literature also facilitates the development of language related to mathematical concept. Rich social interactions with teachers and more competent peers contribute to children's opportunities for learning and developing behavioral knowledge into representational knowledge. The child's ability to use words such as not enough, too many, and so forth would show the highest level of his or her understanding of matching and one-on-one correspondence. It is important for the teacher to recognize that children may put together items in ways that make sense to them, though not necessarily according to the teacher's preconceived ideas. Teachers, therefore, must allow for a wide range of thinking and allow children to explain why they think the way they do.

Fundamental Concept # 2: Early Classification: Creating Sets

Classification is based on the understanding of the concept of "sameness". To group objects together, children need to compare objects and put them in logical groups according to some common criteria. This leads to the creation of a set. Later children begin to recognize that a group of objects is part of a larger group. However it is important to remember that individuals may treat some objects or group of objects as equivalent for different reasons. According to Sugarman (1983) "classification exists when two or more discrete events are treated as equivalent" (p. 4). The ability to classify is considered essential for survival as well as for higher order cognitive processes.

It has been noted that young children (between one and two years of age) will select objects that are the same whether they are prompted to do so or not (Charlesworth & Lind, 1999). They may see relationships between discrete objects, but there is no evidence that they link these objects mentally. The groupings they produce may be generated through a cognitive strategy that involves focussing on individual objects rather than comparing them (Sugarman, 1983). By the age of 3, however, children group objects in ways that suggests some sort of cognitive organization.

Piaget (1965) identified three stages of classification that children experience. Children's initial classifications may be based on incorrect assumptions resulting from the fact that the specifics of the immediate situation rather than all aspects of it captures children's attention. Piaget (1952) called this reasoning transductive and attributed it to children's egocentrism or the ability to perceive objects based only on their own experience with them. This is the stage at which traits such as graphic collections— a random grouping of items— and chaining— placing items in a line focussing on one common attribute between adjacent objects— are evident. In the next stage children begin to use a simple preconceived plan based on a logical, recognizable rationale to classify objects. During this stage the child might make hierarchies in classification. When

children begin to classify in a logical manner they initially form small groups. Later, they form hierarchies of objects that go together or a 'supragroup' (Osborn & Osborn, 1983). The ability to dichotomize is evidence that the child is able to 'see' similarities between and among objects that are apparently dissimilar. The child is thus able to shift criteria and recognize that an object can have numerous attributes and not belong exclusively to one class. The child is thus able to override the dissimilarity and continues to classify according to a preconceived plan (Osborn & Osborn, 1983).

The development of the concepts of Sets & Classification begins with the ability to group identical objects followed by the ability to sort objects by one attribute— color, shape, size, material, pattern and texture, then classify by more abstract attributes like function and association. Sorting objects by color is usually the first step in the developmental sequence. Studies report, however, that approximately 8% of males and 4% of females will have trouble with 'color blindness' (Smith, 1997). Usually creating sets using size (big, little) and shape (round, square) requires an adult's guidance. In the developmental sequence children then classify by two attributes and later by three or more attributes. Class inclusion (joining sets) and class exclusion (separating sets) are the next stages of development. Finally, children also classify by number, for example pairs, groups of three, four etc.

Snack time presents an excellent opportunity for learning classification by two and more attributes, as well as class inclusion. While making a fruit salad, for example, a teacher may ask the children, "We have apples and bananas in this fruit salad; could we add any other fruit?" Clean-up time also provides a number of opportunities to ask children to put *all* the animals in one box or all the paintbrushes in the sink. At each of these levels it is also important that the teacher talks to the children and asks them to describe and explain what has been done. For example, in class inclusion a child might say, "These are all animals, but some of them are dogs" or in class exclusion, "This block is not red so it doesn't go in that group." These types of statements provide excellent opportunities for the teacher to take the conversation further and probe what caused the child's misconception.

Presented below is the checklist based on the developmental sequence of concepts of sets and classifications (Table 2).

Classification of objects is an ability that is demonstrated at a very young age by most children. However, young children do not necessarily know the names of colors, shapes, materials etc. This can be mistaken as a lack of knowledge or ability to classify according to one attribute. Thus the teacher should ask children to classify using general questions such as "Can you find something that is the same color/shape/size/material etc. as this one?" rather than using a specific color or shape. By the time children demonstrate the ability to classify by two or more attributes they have acquired the vocabulary to describe the specific characteristics of the object. It is then appropriate for the teacher to ask the children "Can you find something that is red and long?"

Table 2 Checklist for Preschool Pre-mathematical Concepts: Sets and Classification**Name of Child** _____

Concepts/Stages of Development	Sept-Oct.	Dec.- Jan.	April-May
1. Able to group identical objects			
2. Sorts objects by one attribute – color, shape, size, material, pattern, texture			
3. Classifies by 2 attributes			
4. Classifies by 3 attributes			
5. Describes and/or explains what has been done while classifying by 1,2 or 3 attributes			
6. Classifies according to function			
7. Describes and/or explains what has been done			
Classifies according to association			
8. Describes and/or explains what has been done			
9. Understands class exclusion			
10. Understands class inclusion			
11. Describes and/or explains what has been done			
12. Classifies by number			

The ability to classify by function or association can be demonstrated by the child if the teacher asks questions as “Can you put things that you draw with together?” or “Can you find the things a doctor/hairdresser/gardener etc. uses?” Although it is not typical for preschool children to have a clear understanding of class inclusion and exclusion, when asked specific questions some children might demonstrate partial understanding of the concept. This is particularly true when class inclusion is related to personal experiences as visiting a doctor’s office, going to the grocery store, or gardening with a parent.

In class inclusion, the teacher can provide varied plastic animals and ask the child, “Here are some dogs, can you find some animals to put in this group?” At each of these levels, it is important that the child be able to describe then explain what has been done. The general uses of sets or grouping activities occur often in science class (e.g. desert animals and forest animals), social studies (e.g. city life and farm life), and in number work in math (e.g., making sets of objects, starting with zero and deciding when to stop adding more (Kamii, 2000, Smith, 1997).

Fundamental Concept # 3: Order and Seriation

Ordering, a higher level of comparing, involves comparing more than two things or more than two sets. The earliest comparing concepts are opposites that are readily observed and experienced in the preschool years. Common comparing language includes words as- bigger-smaller, taller-shorter, etc. Ordering or seriation involves putting more than two objects, or sets with more than two members, into a sequence. It also involves placing things in a sequence from first to last. It is a prerequisite for patterning.

Ordering is the foundation of our number system (e.g. two is bigger than one; three is bigger than two, etc.). There are three stages of growth in ordering as described by Piaget (as cited in Copeland, 1984). In the first stage three-to-four-year olds cannot order sticks of various lengths, they place them in random order (so there is no satisfactory pattern). However, young children (2 ½ - 5 years) who attend Montessori School often order wooden cylinders by diameter, placing them in self-correcting compartments, or order sets of color tiles by hue. It would appear that some aspects of ordering tasks improve with practice, and may be accomplished at an early age (Smith, 1997).

Around the age of five, the child can order sticks using trial and error. The sticks are moved from location to location until the task is accomplished. Finally children create order in a systematic fashion. The child of six can consider all the sticks before moving them around, form a plan of operation, and then choose the sticks in a systematic fashion. As the child's ability to order develops she/he uses words (e.g. bigger, smaller, smallest, etc.) while ordering four or more items. Next the child demonstrates the ability to seriate in reverse order and finally to double seriate. Given below is the checklist based on the developmental sequence of the concepts of order and seriation (Table 3).

Table 3
Checklist for Preschool Pre-mathematical Concepts: Order and Seriation

Name of Child _____

Concept/Stages of Development	Sept-Oct.	Dec.- Jan.	April-May
1. Comparison of opposites (e.g., long/short, big/small, etc.)			
2. Orders 3 objects in random order			
3. Orders 3 objects by trial and error			
4. Orders 3 objects in a systematic manner			
5. Seriates in reverse order			
6. Performs double seriation			
7. Describes and/or explains what has been done			

The key to Tables 2 and 3 are the same as for Table 1.

While observing children in the classroom the teacher can use everyday objects to determine children's levels of understanding of the concept of order and seriation. The teacher can observe how a child plays with nesting toys, for example whether they are placed in rows from largest to smallest or smallest to largest. While the child is playing with big and small blocks the teacher could hold up a block and ask a question as, "Can you find something that is smaller than this?" or "Can you find something that is bigger than this block?" While playing with toy vehicles, the teacher could ask the child to park the cars in the garage in order from biggest to smallest or smallest to biggest. A child can demonstrate understanding of seriation by putting characters in his/her painting according to their size or coloring objects according to their hue from lightest to darkest and vice-versa. Seriation can also be demonstrated through children's sequencing of their events while on a field trip. Stories can be used to illustrate understanding of double seriation. The teacher could ask the child to find the biggest bowl for Papa Bear, a medium sized bowl for Mama Bear and the smallest bowl for Baby Bear.

Use of the checklists

As a tool for assessment, the checklists can be used to chart and monitor student performance. The use of the tools can help the teacher identify concepts that are understood by the children, or those that require additional materials and interactions for further development. The teacher provides 'scaffold assistance' which entails a continuous modification of the tasks to provide the appropriate level of challenge that enables the child to learn (Berk & Winsler, 1995). Children learn through meaningful, naturalistic, active learning experiences. The adult must build on their knowledge and take them to higher levels of understanding.

Teachers can use the checklists while observing small groups of children working together or individual children participating in an activity. By watching children and recording their behaviors, teachers can find out how children advance, what they think, and how they feel (Lay-Dopyera & Dopyera, 1993). Observations of young children's naturalistic and informal activities such as getting dressed, setting the table for breakfast, playing with dolls or pots and pans, can and should be the main source of information about very young children's understanding and abilities related to mathematical concepts. For example, the way in which a child spends time lining up the toys and matching items, or reports that more items are needed (e.g. "I don't have enough plates for the table.") are indicators of the child's matching abilities. Accordingly, the way in which a child sorts a collection of toys such as farm animals, reveals his or her level of understanding of classification. By keeping a record of these observations, teachers build up a more comprehensive picture of each individual child's strengths and weaknesses.

A checklist can be used as frequently as the teacher desires to chart development and understanding of concepts. During the first few weeks of the program the teacher can use the checklist to determine the baseline level of understanding. This assessment needs to be performed for all children during free

choice activities while they use a variety of materials. The initial information can help teachers cluster students' performance according to their understanding of fundamental mathematical concepts and use that information as a basis for planning activities and instruction. Periodic and systematic use of the checklists will help monitor the development of the concepts in each child. Dating the observation while using the checklists will provide a record of each child's growth and development, as well as help identify children with similar levels of understanding at any given period of time. This will provide information on which to scaffold the learning process, helping teachers to align curriculum, instruction and assessment.

Conclusion

The checklists provided here should be used to gather information for curriculum development rather than to evaluate or determine mastery. They can be used as a tool to guide curriculum and teaching by providing a developmental sequence in which the three fundamental mathematical concepts are likely to emerge in preschool children. Understanding "how knowledge develops, what they [children] can understand and how they understand their experiences as development proceeds is another basis for curriculum planning" (Katz & Chard (2000, p. 26). When teachers know what mathematical concepts they want children to understand and the stages through which they develop, they can plan meaningful learning experiences and assess children's growth (Richardson & Salkeld, 1995). The checklists presented can assist preschool teachers with regard to providing such experiences for the children with whom they work.

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